L1 –Solving Quadratics by Factoring Unit 5

MPM2D

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You will be able to solve some quadratic equations by:

1) writing them in the form $ax^2 + bx + c = 0$
2) factoring the quadratic
3) setting each factor to zero and solving

Setting each factor to **zero** and solving allows you to find what makes the entire product equal zero because of the zero product rule.

The zero product rule states that the product of factors is zero if one or more of the factors are zero.

For example, if $ab = 0$, then $a = 0$ or $b = 0$ (or both)

For the quadratic equation $x^2 + 2x - 3 = 0$, it factors to $(x + 3)(x - 1) = 0$. Then using the zero product rule, we can see that the product would be zero if either $x + 3 = 0$ or if $x - 1 = 0$. The values of x make those factors become zero are $x = -3$ and $x = 1$. Therefore, those are the solutions to the equation. They can be checked in the original equation.

When trying to factor a quadratic, remember to always check for a common factor. Also, remember that if the leading coefficient in a quadratic trinomial is not equal to 1, you must factor by decomposition.

Example 1: Solve each of the following quadratic equations

a) $x^2 + 8x + 15 = 0$

$5 \times 3 = 15$

$5 + 3 = 8$

$(x+5)(x+3) = 0$

$x+5=0$ $x+3=0$

$x_1 = -5$ $x_2 = -3$

b) $x^2 - 7x = 0$

$x(x-7) = 0$

$x_1 = 0$ $x-7=0$

$x_2 = 7$

c) $x^2 + 4x = 12$

Tip: Always start by setting the quadratic equal to zero.

$x^2+4x-12=0$

$(x+6)(x-2)=0$

$6 \times -2 = -12$

$6 + -2 = 4$

$x+6=0$ $x-2=0$

$x_1 = -6$ $x_2 = 2$

d) $2x^2 - 22x + 48 = 0$

Tip: Always check for a common factor!

$2(x^2-11x+24)=0$

$2(x-8)(x-3)=0$

$-8 \times -3 = 24$

$-8 + -3 = -11$

$x-8=0$ $x-3=0$

$x_1 = 8$ $x_2 = 3$

e) $0 = 2x^2 + 5x - 3$

$6 \times -1 = -6$

$6 + -1 = 5$

$0 = 2x^2 + 6x - 1x - 3$

$0 = 2x(x+3) - 1(x+3)$

$0 = (x+3)(2x-1)$

$x+3 = 0$ $\quad 2x-1 = 0$

$x_1 = -3$ $\quad 2x = 1$

$x_2 = \frac{1}{2}$

f) $3x^2 + 17x = 20$

$3x^2 + 17x - 20 = 0$

$20 \times -3 = -60$

$20 + -3 = 17$

$3x^2 + 20x - 3x - 20 = 0$

$x(3x+20) - 1(3x+20) = 0$

$(3x+20)(x-1) = 0$

$3x+20 = 0$ $\quad x-1 = 0$

$3x = -20$ $\quad x_2 = 1$

$x_1 = -\frac{20}{3}$

g) $2x^2 - 6 = 0$

$2x^2 = 6$

$x^2 = \frac{6}{2}$

$x^2 = 3$

$x = \pm\sqrt{3}$

h) $3x^2 + 1 = 0$

$3x^2 = -1$

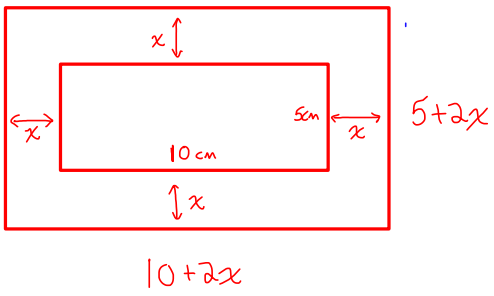
$x^2 = -\frac{1}{3}$

$x = \pm\sqrt{-\frac{1}{3}}$

so there are NO real solutions

Note: these are irrational solutions

Example 2: A picture that measures 10 cm by 5 cm is to be surrounded by a mat before being framed. The width of the mat is to be the same on all sides of the picture. The area of the mat is to be twice the area of the picture. What is the width of the mat?



Area of mat = 2 (area of picture)

$(10+2x)(5+2x) - 50 = 2(5)(10)$

$50 + 20x + 10x + 4x^2 - 50 = 100$

$4x^2 + 30x - 100 = 0$

$2(2x^2 + 15x - 50) = 0$

$20 \times -5 = -100$

$20 + -5 = 15$

$2(2x^2 + 20x - 5x - 50) = 0$

$2[2x(x+10) - 5(x+10)] = 0$

$2(x+10)(2x-5) = 0$

$x+10 = 0$ $\quad 2x-5 = 0$

$x = -10$ $\quad 2x = 5$

reject answer $\quad x = \frac{5}{2}$

The width of the mat is 2.5 cm