L3 –Solving Quadratics using the Quadratic Formula Unit 5
MPM2D
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Part 1: Proof of Quadratic Formula

The quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

is a formula that can be used to find rational, irrational, and non-real solutions to any quadratic equation that is in the form $ax^2 + bx + c = 0$. It is a formula that is derived from completing the square of a general standard form quadratic and then rearranging to isolate for x. Using the formula is just a shortcut to avoid having to complete the square and rearrange like we did last lesson.

Proof:

Part 2: Rational Solutions

Example 2: Solve each of the following equations using the Quadratic Formula.

a) $0 = x^2 - 7x + 12$

b) $0 = 3x^2 + 11x - 4$

c) $-9 = x^2 - 6x$

Notice for each of the equations in this question there were ___________________ solutions. That means that each of the equations could have been solved by factoring. But quadratic formula also works!

Part 3: Irrational Solutions

Example 3: Solve each of the following quadratic equations

a) $0 = 3x^2 + 8x - 5$

b) $-5 = -x^2 - 10x$

Notice for each of the equations in this question there were only ___________________ solutions. That means that each of the equations could NOT have been solved by factoring. This is why the Quadratic formula is so useful!

Part 4: Non-Real Solutions

Example 4: Solve each of the following quadratic equations

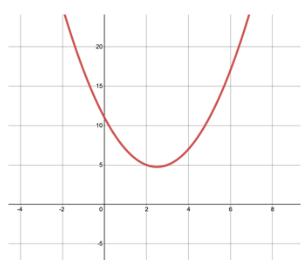
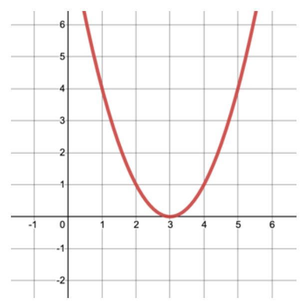
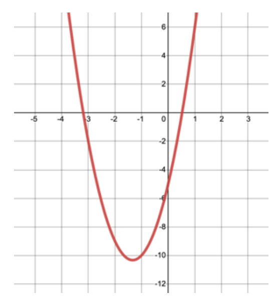
a) $0 = x^2 - 5x + 11$

b) $-7 = 2x^2 - 4x$

There are no real solutions when under the square root (the discriminant) is ___________________. You can't square root a negative!

Part 5: The Discriminant

Looking back through the previous parts of the lesson, you can see that quadratic equations can have either zero, one, or two solutions. The discriminant, $b^2 - 4ac$, is the part of the equation that tells you how many solutions a quadratic equation will have.

# of solutions	0	1	2
Discriminant	$b^2 - 4ac < 0$	$b^2 - 4ac = 0$	$b^2 - 4ac > 0$
Example Equation	$0 = x^2 - 5x + 11$ $x = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(11)}}{2(1)}$ $x = \frac{5 \pm \sqrt{-19}}{2}$ No real solutions Notice $b^2 - 4ac = -19$ which is less than zero	$0 = x^2 - 6x + 9$ $x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(9)}}{2(1)}$ $x = \frac{6 \pm \sqrt{0}}{2(1)}$ $x = 3$ Notice $b^2 - 4ac = 0$	$0 = 3x^2 + 8x - 5$ $x = \frac{-8 \pm \sqrt{(8)^2 - 4(3)(-5)}}{2(3)}$ $x = \frac{-8 \pm \sqrt{124}}{6}$ $x = -3.19, 0.52$ Notice $b^2 - 4ac = 124$ which is greater than zero
Example Graph	 Notice the graph never crosses the x-axis	 Notice the vertex is ON the x-axis	 Notice the graph crosses through the x-axis twice

Example 5: How many real solutions does each quadratic equation have?

a) $0 = x^2 + 10x + 25$ **b)** $0 = -2x^2 + 2x + 7$ **c)** $0 = 2x^2 + 2x + 7$

Another piece of useful information about the discriminant:

If $b^2 - 4ac$ is a ____________________________ number, you will get rational solutions which means the quadratic is factorable. If you aren't sure if a quadratic is factorable, just check to see if $b^2 - 4ac$ is a perfect square number $(0, 1, 4, 9, 16, 25, 36, \ldots)$