Part 1: Vertex from Standard Form Quadratic

Remember that parabolas are symmetrical about the axis of symmetry which is a vertical line that passes through the vertex. Because of this symmetry property, you can find the x-coordinate of the vertex by averaging the x-intercepts.

From quadratic formula we know that the x-intercepts of a standard form quadratic are

$$x = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } x = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Therefore, the x-coordinate of the vertex is:

$$x\text{-vertex} = \frac{\left(\frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a}\right)}{2}$$

$$= \frac{\left(\frac{-2b}{2a}\right)}{\left(\frac{2}{1}\right)}$$

$$= \frac{-b}{a} \times \frac{1}{2}$$

$$= \frac{-b}{2a}$$

Conclusion:

From the standard form equation of a quadratic, $y = ax^2 + bx + c$, you can determine the x-coordinate of the vertex using the formula:

$$x - vertex = \frac{-b}{2a}$$

Example 1: Find the vertex of the following quadratics

a) $y = x^2 - 6x + 11$

$$x\text{-vertex} = \frac{6}{2(1)} = 3$$

$$y\text{-vertex} = (3)^2 - 6(3) + 11$$
$$= 2$$

$$\boxed{(3, 2)}$$

b) $y = -3x^2 + 2x - 1$

$$x\text{-vertex} = \frac{-2}{2(-3)} = \frac{-2}{-6} = \frac{1}{3}$$

$$y\text{-vertex} = -3\left(\frac{1}{3}\right)^2 + 2\left(\frac{1}{3}\right) - 1$$
$$= -\frac{2}{3}$$

$$\boxed{\left(\frac{1}{3}, -\frac{2}{3}\right)}$$

Part 2: Putting it all together

Example 2: For the quadratic $y = -5x^2 + 8x - 3$

a) Find the x-intercepts
b) Find the axis of symmetry
c) Find the vertex
d) Sketch the graph labelling key points

a) $0 = -5x^2 + 8x - 3$

$\underline{5} \times \underline{3} = 15$

$\underline{5} + \underline{3} = 8$

$0 = -5x^2 + 5x + 3x - 3$

$0 = -5x(x-1) + 3(x-1)$

$0 = (x-1)(-5x+3)$

$x - 1 = 0$

$\boxed{x = 1}$

$-5x + 3 = 0$

$3 = 5x$

$\boxed{\frac{3}{5} = x}$

b) aos: $x = \frac{1 + 0.6}{2}$

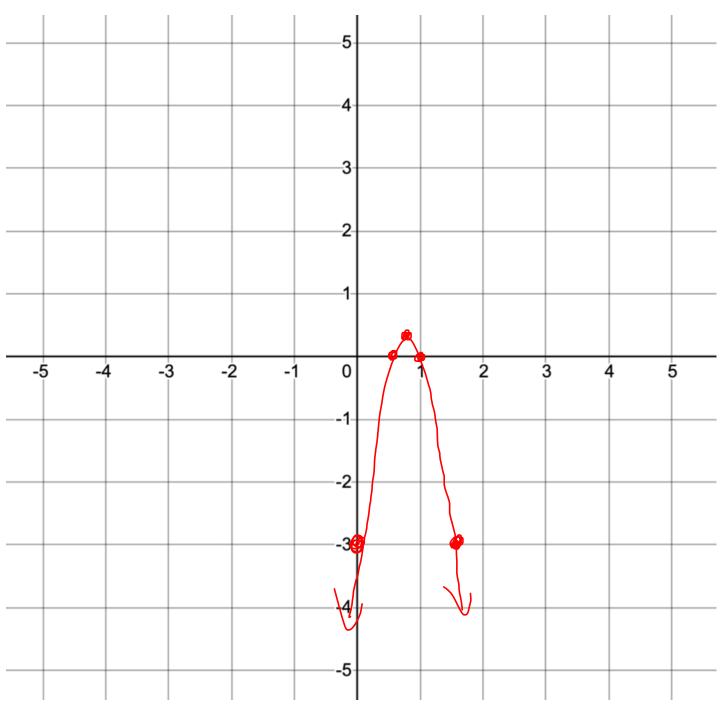
$\boxed{x = 0.8}$

c) x-vertex $= 0.8$

y-vertex $= -5(0.8)^2 + 8(0.8) - 3$

$= 0.2$

$\boxed{(0.8, 0.2)}$



Example 3: For the quadratic $y = 2x^2 - 8x + 11$

a) Find the x-intercepts
b) Find the axis of symmetry
c) Find the vertex
d) Sketch the graph labelling key points

a) $0 = 2x^2 - 8x + 11$

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(2)(11)}}{2(2)}$$

$$x = \frac{8 \pm \sqrt{-24}}{4}$$

$\therefore$ no x-intercepts

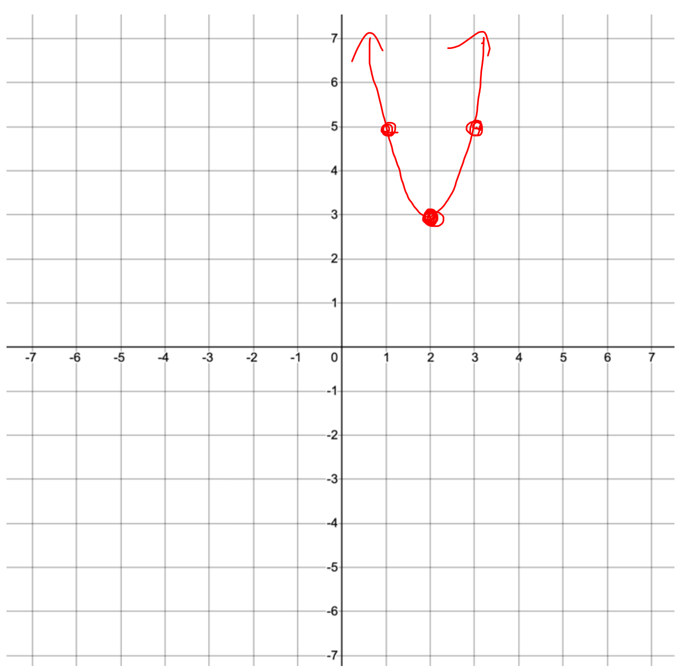
b) aos: $x = \frac{8}{2(2)}$

$x = 2$

c) x-vertex $= 2$

y-vertex $= 2(2)^2 - 8(2) + 11$
$= 3$

$(2, 3)$



Example 4: For the quadratic $y = x^2 - 10x + 25$

a) Find the x-intercepts
b) Find the axis of symmetry
c) Find the vertex
d) Sketch the graph labelling key points

a) $0 = x^2 - 10x + 25$

$0 = (x-5)^2$

$x - 5 = 0$

$\boxed{x = 5}$

$\underline{-5} \times \underline{-5} = 25$

$\underline{-5} + \underline{-5} = -10$

b) aos: $x = \dfrac{10}{2(1)}$

$\boxed{x = 5}$

c) x-vertex $= 5$

y-vertex $= (5)^2 - 10(5) + 25$
$= 0$

$\boxed{(5, 0)}$

Note: when a quadratic only has 1 x-intercept, the vertex must be on the x-axis.