

W1 –Solving Quadratics by Factoring

Unit 5

MPM2D

Jensen

1) Solve

a) $(x + 1)(x + 2) = 0$

$$\begin{aligned} x+1 &= 0 & x+2 &= 0 \\ x_1 &= -1 & x_2 &= -2 \end{aligned}$$

b) $(x + 3)(x - 1) = 0$

$$\begin{aligned} x+3 &= 0 & x-1 &= 0 \\ x_1 &= -3 & x_2 &= 1 \end{aligned}$$

c) $x(4 - x) = 0$

$$\begin{aligned} x_1 &= 0 & 4-x &= 0 \\ & & 4 &= x_2 \end{aligned}$$

d) $(2x + 1)(x - 3) = 0$

$$\begin{aligned} 2x+1 &= 0 & x-3 &= 0 \\ 2x &= -1 & x_2 &= 3 \\ x_1 &= -\frac{1}{2} & & \end{aligned}$$

2) Solve and check

a) $x^2 + 7x + 12 = 0$

$$\begin{aligned} &\frac{4}{4} \times \frac{3}{3} = 12 \\ (x+4)(x+3) &= 0 \quad \frac{4}{4} + \frac{3}{3} = 7 \end{aligned}$$

$$\begin{aligned} x+4 &= 0 & x+3 &= 0 \\ x_1 &= -4 & x_2 &= -3 \end{aligned}$$

b) $x^2 - x - 6 = 0$

$$\begin{aligned} &\frac{-3}{-3} \times \frac{2}{2} = -6 \\ (x-3)(x+2) &= 0 \quad \frac{-3}{-3} + \frac{2}{2} = -1 \end{aligned}$$

$$\begin{aligned} x-3 &= 0 & x+2 &= 0 \\ x_1 &= 3 & x_2 &= -2 \end{aligned}$$

c) $x^2 - 8x + 16 = 0$

$$\begin{aligned} &\frac{-4}{-4} \times \frac{-4}{-4} = 16 \\ (x-4)(x-4) &= 0 \quad \frac{-4}{-4} + \frac{-4}{-4} = -8 \\ (x-4)^2 &= 0 \end{aligned}$$

$$\begin{aligned} x-4 &= 0 \\ x &= 4 \end{aligned}$$

f) $x^2 - 7x = 18$

$$\begin{aligned} x^2 - 7x - 18 &= 0 \quad \frac{-9}{-9} \times \frac{2}{2} = -18 \\ (x-9)(x+2) &= 0 \quad \frac{-9}{-9} + \frac{2}{2} = -7 \end{aligned}$$

$$\begin{aligned} x-9 &= 0 & x+2 &= 0 \\ x_1 &= 9 & x_2 &= -2 \end{aligned}$$

3) Solve

a) $2a^2 + 3a - 2 = 0$ $\frac{4}{4} \times \frac{-1}{-1} = -4$

$$2a^2 + 4a - 1a - 2 = 0$$

$$2a(a+2) - 1(a+2) = 0$$

$$(a+2)(2a-1) = 0$$

$$a+2=0 \quad 2a-1=0$$

$$a_1 = -2$$

$$2a=1$$

$$a_2 = \frac{1}{2}$$

c) $2t^2 + 11t + 5 = 0$ $\frac{10}{10} \times \frac{1}{1} = 10$

$$2t^2 + 10t + t + 5 = 0$$

$$2t(t+5) + 1(t+5) = 0$$

$$(t+5)(2t+1) = 0$$

$$t+5=0 \quad 2t+1=0$$

$$t_1 = -5$$

$$2t=-1$$

$$t_2 = -\frac{1}{2}$$

e) $3 = 4m^2 - 4m$ $\frac{-6}{-6} \times \frac{2}{2} = -12$

$$0 = 4m^2 - 4m - 3$$

$$0 = 4m^2 - 6m + 2m - 3$$

$$0 = 2m(2m-3) + 1(2m-3)$$

$$0 = (2m-3)(2m+1)$$

$$2m-3=0 \quad 2m+1=0$$

$$2m=3$$

$$m_1 = \frac{3}{2}$$

$$2m=-1$$

$$m_2 = -\frac{1}{2}$$

g) $x^2 + 2x = 0$

$$x(x+2) = 0$$

$$x_1 = 0$$

$$x+2=0$$

$$x_2 = -2$$

b) $3s^2 - 4s + 1 = 0$ $\frac{-3}{-3} \times \frac{-1}{-1} = 3$

$$3s^2 - 3s - 1s + 1 = 0$$

$$3s(s-1) - 1(s-1) = 0$$

$$(s-1)(3s-1) = 0$$

$$s-1=0 \quad 3s-1=0$$

$$s_1 = 1$$

$$3s=1$$

$$s_2 = \frac{1}{3}$$

d) $3x^2 + 7x - 6 = 0$ $\frac{9}{9} \times \frac{-2}{-2} = -18$

$$3x^2 + 9x - 2x - 6 = 0$$

$$3x(x+3) - 2(x+3) = 0$$

$$(x+3)(3x-2) = 0$$

$$x+3=0 \quad 3x-2=0$$

$$x_1 = -3$$

$$3x=2$$

$$x_2 = \frac{2}{3}$$

f) $10y^2 - 16y = -6$

$$10y^2 - 16y + 6 = 0$$

$$2(5y^2 - 8y + 3) = 0$$

$$5y^2 - 5y - 3y + 3 = 0$$

$$5y(y-1) - 3(y-1) = 0$$

$$(y-1)(5y-3) = 0$$

$$y-1=0$$

$$5y-3=0$$

$$5y=3$$

h) $3x^2 + 2x = 0$

$$x(3x+2) = 0$$

$$x_1 = 0$$

$$3x+2=0$$

$$3x = -2$$

$$x_2 = -\frac{2}{3}$$

$$i) 5x^2 - 20x = 0$$

$$5x(x-4) = 0$$

$$x_1 = 0$$

$$x-4 = 0$$

$$x_2 = 4$$

$$j) 0 = 4x + 3x^2$$

$$0 = x(4+3x)$$

$$x_1 = 0$$

$$4+3x = 0$$

$$3x = -4$$

$$x_2 = \frac{-4}{3}$$

$$k) x^2 - 25 = 0$$

$$(x-5)(x+5) = 0$$

$$x-5 = 0 \quad x+5 = 0$$

$$x_1 = 5$$

$$x_2 = -5$$

$$l) x^2 + 4 = 16$$

$$x^2 = 12$$

$$x = \pm\sqrt{12}$$

$$x = \pm 2\sqrt{3}$$

$$x \approx \pm 3.46$$

$$m) x^2 - 2x - 11 = 4$$

$$x^2 - 2x - 15 = 0 \quad \frac{-5}{-5} \times \frac{3}{3} = -15$$

$$(x-5)(x+3) = 0$$

$$x-5 = 0 \quad x+3 = 0$$

$$x_1 = 5$$

$$x_2 = -3$$

$$n) 5x^2 = 2x$$

$$5x^2 - 2x = 0$$

$$x(5x-2) = 0$$

$$x_1 = 0 \quad 5x-2 = 0$$

$$5x = 2$$

$$x_2 = \frac{2}{5}$$

$$o) (x+4)^2 = 4$$

$$x+4 = \pm\sqrt{4}$$

$$x+4 = \pm 2$$

$$x+4 = 2$$

$$x+4 = -2$$

$$x_1 = -2$$

$$x_2 = -6$$

$$p) (x-6)^2 - 8x = 0$$

$$x^2 - 12x + 36 - 8x = 0$$

$$x^2 - 20x + 36 = 0$$

$$(x-2)(x-18) = 0$$

$$x_1 = 2 \quad x_2 = 18$$

$$\frac{-2}{-2} \times \frac{18}{18} = 36$$

$$\frac{-2}{-2} + \frac{-18}{-18} = -20$$

4) The area of the rectangle shown in the diagram is 36 cm^2 . What are its dimensions?

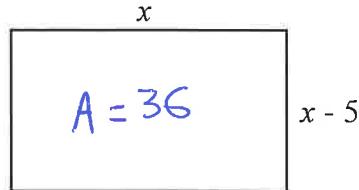
$$x(x-5) = 36$$

$$x^2 - 5x - 36 = 0 \quad \frac{-9}{-9} \times \frac{4}{4} = -36$$

$$(x-9)(x+4) = 0 \quad \frac{-9}{-9} + \frac{4}{4} = -5$$

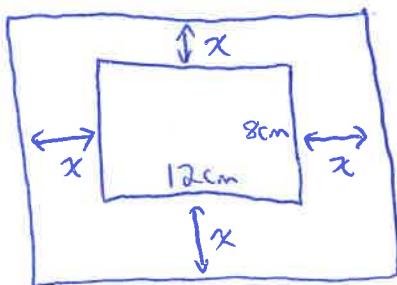
$$x-9=0 \quad x+4=0$$

$$x_1=9 \quad \begin{array}{l} x_2=-4 \\ \text{reject} \end{array}$$



The dimensions are 9cm and 4cm.

5) A photograph measuring 12 cm by 8 cm is to be surrounded by a mat before framing. The width of the mat is to be the same on all sides of the photograph. The area of the mat is to equal the area of the photograph. Find the width of the mat.



$$\text{Area of mat} = \text{Area of photo}$$

$$(12+2x)(8+2x) - 12(8) = 12(8)$$

$$96 + 24x + 16x + 4x^2 - 96 = 96$$

$$4x^2 + 40x - 96 = 0$$

~~$$4(x^2 + 10x - 24) = 0$$~~

$$4(x+12)(x-2) = 0$$

$$x+12=0 \quad x-2=0$$

$$\begin{array}{l} x=-12 \quad x=2 \\ \text{reject} \end{array}$$

$$\begin{array}{l} 12 \quad x-2 = -24 \\ 12 + -2 = 10 \end{array}$$

The width is 2cm.

6) Three times the square of an integer is 432. Find the integer.

$$3x^2 = 432$$

$$x^2 = 144$$

$$x = \pm \sqrt{144}$$

$$x = \pm 12$$

- 7) A regular polygon with n sides has $\frac{n(n-3)}{2}$ diagonals. Find the number of sides of a regular polygon that has 44 diagonals.

$$(2) 44 = \frac{n(n-3)}{2} (\cancel{x})$$

$$88 = n^2 - 3n$$

$$0 = n^2 - 3n - 88 \quad -11 \times 8 = -88$$

$$0 = (n-11)(n+8) \quad -11 + 8 = -3$$

$$n-11=0 \quad n+8=0$$

$$\textcircled{n_1 = 11}$$

$$n_2 = -8$$

reject

11 sides

Answers

- 1)a) $x = -2, -1$ b) $x = -3, 1$ c) $x = 0, 4$ d) $x = -\frac{1}{2}, 3$
- 2)a) $x = -4, -3$ b) $x = -2, 3$ c) $x = 4$ d) $x = -2, 9$
- 3)a) $a = -2, \frac{1}{2}$ b) $s = \frac{1}{3}, 1$ c) $t = -5, -\frac{1}{2}$ d) $x = -3, \frac{2}{3}$ e) $m = -\frac{1}{2}, \frac{3}{2}$ f) $y = \frac{3}{5}, 1$ g) $x = -2, 0$
- h) $x = -\frac{2}{3}, 0$ i) $x = 0, 4$ j) $x = -\frac{4}{3}, 0$ k) $x = -5, 5$ l) $x = \pm\sqrt{12} = \pm 2\sqrt{3} \cong \pm 3.46$ m) $x = -3, 5$
- n) $x = 0, \frac{2}{5}$ o) $x = -6, -2$ p) $x = 2, 18$
- 4) 9 cm by 4 cm
- 5) 2 cm
- 6) 12 or -12
- 7) 11