W3 –Solving Quadratics using the Quadratic Formula Unit 5
MPM2D
Jensen

1) Use the quadratic formula to solve each equation. Express answers as ~~exact~~ approximate. roots.

a) $7x^2 + 24x + 9 = 0$

$x = \frac{-24 \pm \sqrt{(24)^2 - 4(7)(9)}}{2(7)}$

$x = \frac{-24 \pm \sqrt{324}}{14}$

$x = \frac{-24+18}{14}$ $x = \frac{-24-18}{14}$

$x_1 = -\frac{3}{7}$ $x_2 = -3$

b) $2x^2 + 4x - 7 = 0$

$x = \frac{-4 \pm \sqrt{(4)^2 - 4(2)(-7)}}{2(2)}$

$x = \frac{-4 \pm \sqrt{72}}{4}$

$x = \frac{-4+\sqrt{72}}{4}$ $x = \frac{-4-\sqrt{72}}{4}$

$x_1 \simeq 1.12$ $x_2 \simeq -3.12$

c) $4x^2 - 12x + 9 = 0$

$x = \frac{12 \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)}$

$x = \frac{12 \pm \sqrt{0}}{8}$

$x = \frac{12}{8}$

$x = \frac{3}{2}$

d) $2x^2 - 7x = -4$

$2x^2 - 7x + 4 = 0$

$x = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(4)}}{2(2)}$

$x = \frac{7 \pm \sqrt{17}}{4}$

$x = \frac{7+\sqrt{17}}{4}$ $x = \frac{7-\sqrt{17}}{4}$

$x_1 \simeq 2.78$ $x_2 \simeq 0.72$

e) $3x^2 + 5x = 1$

$3x^2 + 5x - 1 = 0$

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-1)}}{2(3)}$

$x = \frac{-5 \pm \sqrt{37}}{6}$

$x_2 = \frac{-5+\sqrt{37}}{6}$ $x_2 = \frac{-5-\sqrt{37}}{6}$

$x_1 \simeq 0.18$ $x_2 \simeq -1.85$

f) $16x^2 + 24x = -9$

$16x^2 + 24x + 9 = 0$

$x = \frac{-24 \pm \sqrt{(24)^2 - 4(16)(9)}}{2(16)}$

$x = \frac{-24 \pm \sqrt{0}}{32}$

$x = \frac{-24}{32}$

$x = -\frac{3}{4}$

2) Use the quadratic formula to solve. Express your answers ~~as exact roots and~~ as approximate roots, rounded to the nearest hundredth.

a) $3x^2 + 14x + 5 = 0$

$$x = \frac{-14 \pm \sqrt{(14)^2 - 4(3)(5)}}{2(3)}$$

$$x = \frac{-14 \pm \sqrt{136}}{6}$$

$$x_1 = \frac{-14 + \sqrt{136}}{6} \qquad x_2 = \frac{-14 - \sqrt{136}}{6}$$

$$\boxed{x_1 \simeq -0.39} \qquad \boxed{x_2 \simeq -4.28}$$

b) $8x^2 + 12x + 1 = 0$

$$x = \frac{-12 \pm \sqrt{(12)^2 - 4(8)(1)}}{2(8)}$$

$$x = \frac{-12 \pm \sqrt{112}}{16}$$

$$x_1 = \frac{-12 + \sqrt{112}}{16} \qquad x_2 = \frac{-12 - \sqrt{112}}{16}$$

$$\boxed{x_1 \simeq -0.09} \qquad \boxed{x_2 \simeq -1.41}$$

c) $4x^2 - 7x - 1 = 0$

$$x = \frac{7 \pm \sqrt{(-7)^2 - 4(4)(-1)}}{2(4)}$$

$$x = \frac{7 \pm \sqrt{65}}{8}$$

$$x_1 = \frac{7 + \sqrt{65}}{8} \qquad x_2 = \frac{7 - \sqrt{65}}{8}$$

$$\boxed{x_1 \simeq 1.88} \qquad \boxed{x_2 \simeq -0.13}$$

d) $10x^2 - 45x - 7 = 0$

$$x = \frac{45 \pm \sqrt{(-45)^2 - 4(10)(-7)}}{2(10)}$$

$$x = \frac{45 \pm \sqrt{2305}}{20}$$

$$x_1 = \frac{45 + \sqrt{2305}}{20} \qquad x_2 = \frac{45 - \sqrt{2305}}{20}$$

$$\boxed{x_1 \simeq 4.65} \qquad \boxed{x_2 \simeq -0.15}$$

e) $-5x^2 + 16x - 2 = 0$

$$x = \frac{-16 \pm \sqrt{(16)^2 - 4(-5)(-2)}}{2(-5)}$$

$$x = \frac{-16 \pm \sqrt{216}}{-10}$$

$$x_1 = \frac{-16 + \sqrt{216}}{-10} \qquad x_2 = \frac{-16 - \sqrt{216}}{-10}$$

$$\boxed{x_1 \simeq 0.13} \qquad \boxed{x_2 \simeq 3.07}$$

f) $-6x^2 + 17x + 5 = 0$

$$x = \frac{-17 \pm \sqrt{(17)^2 - 4(-6)(5)}}{2(-6)}$$

$$x = \frac{-17 \pm \sqrt{409}}{-12}$$

$$x_1 = \frac{-17 + \sqrt{409}}{-12} \qquad x_2 = \frac{-17 - \sqrt{409}}{-12}$$

$$\boxed{x_1 \simeq -0.27} \qquad \boxed{x_2 \simeq 3.1}$$

g) $x^2 + 5x + 2 = 0$

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(1)(2)}}{2(1)}$

$x = \frac{-5 \pm \sqrt{17}}{2}$

$x_1 = \frac{-5 + \sqrt{17}}{2}$ $x_2 = \frac{-5 - \sqrt{17}}{2}$

$x_1 \simeq -0.44$ $x_2 \simeq -4.56$

h) $5x^2 - 3x + 2 = 0$

$x = \frac{3 \pm \sqrt{(-3)^2 - 4(5)(2)}}{2(5)}$

$x = \frac{3 \pm \sqrt{-31}}{10}$

∴ no real solutions

3) Describe the roots of the equation $ax^2 + bx + c = 0$ in each of the following situations. Explain and justify your reasoning, and give examples to support your answers.

a) $b^2 - 4ac < 0$

No real solutions. The square root of a negative number is NOT a real number. You get no real solutions if the quadratic opens up and has its vertex above the x-axis OR if the quadratic opens down and has its vertex below the x-axis.

b) $b^2 - 4ac = 0$

1 real solution. In the QF, adding and subtracting 0 gives the same result. You get 1 solution when the vertex is ON the x-axis.

c) $b^2 - 4ac > 0$ and is a perfect square

You get 2 solutions that are rational numbers. If this happens, solving by factoring would also work.

d) $b^2 - 4ac > 0$ and is NOT a perfect square

You get 2 solutions that are irrational numbers. If this happens, solving by factoring would NOT work. QF must be used.

4) Use the discriminant to determine the number of roots for each quadratic equation.

a) $x^2 - 10x + 25 = 0$

$b^2 - 4ac = (-10)^2 - 4(1)(25)$
$= 0$

$\therefore$ 1 solution

b) $3x^2 + 4x + \frac{4}{3} = 0$

$b^2 - 4ac = (4)^2 - 4(3)(\frac{4}{3})$
$= 0$

$\therefore$ 1 solution

c) $2x^2 - 8x + 9 = 0$

$b^2 - 4ac = (-8)^2 - 4(2)(9)$
$= -8$

$\therefore$ no real solutions

d) $-2x^2 + 0.75x + 5 = 0$

$b^2 - 4ac = (0.75)^2 - 4(-2)(5)$
$= 40.5625$

$\therefore$ 2 solutions.

Answers

1)a) $-3, \frac{-3}{7}$ **b)** $\frac{-4\pm\sqrt{72}}{4} = \frac{-2\pm3\sqrt{2}}{2}$ **c)** $\frac{3}{2}$ **d)** $\frac{7\pm\sqrt{17}}{4}$ **e)** $\frac{-5\pm\sqrt{37}}{6}$ **f)** $\frac{-3}{4}$

2)a) $\frac{-7\pm\sqrt{34}}{3}; -0.39, -4.28$ **b)** $\frac{-3\pm\sqrt{7}}{4}; -0.09, -1.41$ **c)** $\frac{7\pm\sqrt{65}}{8}; 1.88, -0.13$

d) $\frac{45\pm\sqrt{2305}}{20}; 4.65, -0.15$ **e)** $\frac{-16\pm\sqrt{216}}{-10} = \frac{8\pm3\sqrt{6}}{5}; 0.13, 3.07$ **f)** $\frac{17\pm\sqrt{409}}{12}; 3.1, -0.27$

g) $\frac{-5\pm\sqrt{17}}{2}; -0.44, -4.56$ **h)** no real solutions

3)a) no real solutions **b)** 1 real solution **c)** 2 real rational solutions **d)** 2 real irrational solutions

4)a) one **b)** one **c)** none **d)** two