W5 – Applications of Quadratics Unit 5
MPM2D
Jensen

1) A model rocket is launched from the deck in Jim's backyard and the path followed by the rocket can be modelled by the relation $h = -5t^2 + 100t + 15$, where h, in meters, is the height that the model rocket reaches after t seconds.

a) What is the height of the deck?

$h = -5(0)^2 + 100(0) + 15$

$h = 15\,m$

b) What is the height of the model rocket after 2 s?

$h = -5(2)^2 + 100(2) + 15 = 195\,m$

c) What is the maximum height reached by the model rocket?

$x\text{-vertex} = \frac{-100}{2(-5)} = 10$ seconds

$y\text{-vertex} = -5(10)^2 + 100(10) + 15 = 515\,m$

the max height is 515 m.

d) How long did the model rocket take to reach this height?

10 seconds

e) How long was the model rocket above 200 m?

$200 = -5t^2 + 100t + 15$

$0 = -5t^2 + 100t - 185$

$0 = -5(t^2 - 20t + 37)$

$0 = t^2 - 20t + 37$

$t = \frac{20 \pm \sqrt{(-20)^2 - 4(1)(37)}}{2(1)}$

$t = \frac{20 \pm \sqrt{252}}{2}$

$t = 2.06, 17.94$

Time above 200m = $17.94 - 2.06 = \boxed{15.885}$

f) How long the model rocket was in the air.

$0 = -5t^2 + 100t + 15$

$0 = -5(t^2 - 20t - 3)$

$0 = t^2 - 20t - 3$

$t = \frac{20 \pm \sqrt{(-20)^2 - 4(1)(-3)}}{2(1)}$

$t = \frac{20 \pm \sqrt{412}}{2}$

$t \approx \cancel{-0.15}$ (reject), 20.15

It was in the air for 20.15 seconds

2) A harbour ferry service has about 240 000 riders per day for a fare of $2. The port authority wants to increase the fare to help with increasing operational costs. Research has shown that for every $0.10 increase in the fare the number of riders will drop by 10 000. ~~The port authority established a relation defined by $R = -1000p^2 + 4000p + 480000$, where R represents the revenue from fares and p represents the number of $0.10 increases in the fare.~~

a) What increase in the fare will maximize the revenue?

let n = # of $0.10 price increases

$R = (\text{cost})(\#\text{sold})$

$R = (2 + 0.1n)(240000 - 10000n)$

$0 = (2 + 0.1n)(240000 - 10000n)$

$0 = (2 + 0.1n)$ $\quad$ $0 = (240000 - 10000n)$

$-2 = 0.1n$ $\quad$ $10\,000n = 240\,000$

$\frac{-2}{0.1} = n$ $\quad$ $n = \frac{240\,000}{10\,000}$

$n = -20$ $\quad$ $n = 24$

x-vertex $= \frac{-20 + 24}{2} = 2$ price increases

y-vertex $= [2 + 0.1(2)][240000 - 10000(2)]$

$= (2.2)(220000)$

$= \$484000$

To maximize revenues, they should increase the price by $0.20 up to $2.20 per ticket.

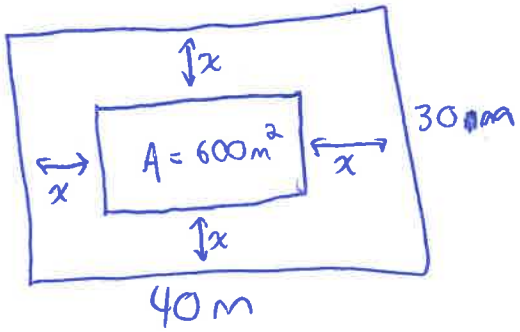
b) What is the new fare?

$220.

c) What is the revenue that will be received from the new fare?

$484000

3) A rectangular lawn measures 30 m by 40 m. Jason is cutting the lawn from the outside perimeter in toward the center by cutting strips along the entire perimeter first, then continuing as he cuts toward the center. How wide is the strip that has been cut along the outside when the area is half cut?



$(30-2x)(40-2x) = \frac{1}{2}(30)(40)$

$1200-60x-80x+4x^2 = 600$

$4x^2-140x+600 = 0$

$4(x^2-35x+150) = 0$

$x^2-35x+150 = 0$

$(x-5)(x-30) = 0$

$x-5=0 \quad x-30=0$

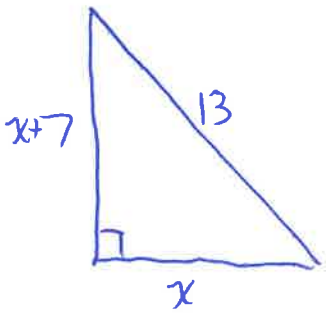
$x=5 \quad \cancel{x=30}$

reject

domain: $0<x<15$

The strip is 5m wide.

4) The hypotenuse of a right triangle measures 13 cm. The legs of the triangle differ by 7 cm. Find the length of each leg.



$(x)^2 + (x+7)^2 = 13^2$

$x^2 + x^2 + 14x + 49 = 169$

$2x^2 + 14x - 120 = 0$

$2(x^2 + 7x - 60) = 0$

$x^2 + 7x - 60 = 0$

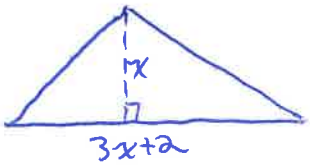
$(x+12)(x-5) = 0$

$x+12 = 0 \quad x-5 = 0$

~~$x = -12$~~ $\quad x = 5$

The legs are 5cm and 12cm

5) A triangle has an area of 308 cm^2. If the base is 2 cm more than three times the height of the triangle, find the base and height of the triangle.



height $= x$

base $= 3x+2$

$A = \frac{1}{2}bh$

$308 = \frac{1}{2}(3x+2)(x)$

$308 = \frac{3}{2}x^2 + x$

$0 = \frac{3}{2}x^2 + x - 308$

$0 = \frac{1}{2}(3x^2 + 2x - 616)$

$0 = 3x^2 + 2x - 616$

$x = \frac{-2 \pm \sqrt{(2)^2 - 4(3)(-616)}}{2(3)}$

$x = \frac{-2 \pm 86}{6}$

$x = -14, 14$

The dimensions are:

$h = 14$ cm

$b = 3(14)+2 = 44$ cm

6) The sum of the squares of four consecutive integers is 630. Find the integers.

$(x)^2 + (x+1)^2 + (x+2)^2 + (x+3)^2 = 630$

$x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 + x^2 + 6x + 9 = 630$

$4x^2 + 12x - 616 = 0$

$4(x^2 + 3x - 154) = 0$

$x^2 + 3x - 154 = 0$

$(x+14)(x-11) = 0$

$x+14 = 0 \quad x-11 = 0$

$x = -14 \quad x = 11$

The integers are either

$-14, -13, -12, -11$

OR

$11, 12, 13, 14$

7) Twice the width of a rectangle is 3 m more than the length. If the area of the rectangle is 209 m^2, find the dimensions of the rectangle.

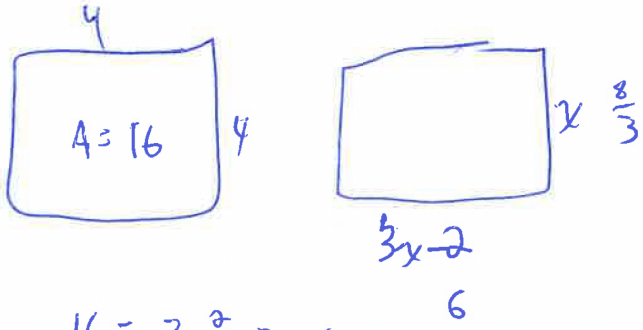
length $= 2x - 3$
width $= x$

$209 = x(2x-3)$
$209 = 2x^2 - 3x$
$0 = 2x^2 - 3x - 209$
$0 = 2x^2 - 22x + 19x - 209$
$0 = 2x(x-11) + 19(x-11)$
$0 = (x-11)(2x+19)$
$x - 11 = 0 \quad 2x + 19 = 0$
$x = 11 \quad x = -9.5$

$A = 209\ m^2$ x width
$2x-3$ length

The dimensions are:
width $= 11$ m
length $= 2(11) - 3 = 19$ m

8) A rectangular carpet and a square carpet have equal areas. The square carpet has a side length of 4 meters. The length of the rectangular carpet is 2 meters less than three times its **w**idth. Find the dimensions of the rectangular carpet.



$16 = 3x^2 - 2x$
$0 = 3x^2 - 2x - 16$
$0 = 3x^2 - 8x + 6x - 16$
$0 = x(3x-8) + 2(3x-8)$

Answers

1)a)15 m **b)**195 m **c)** 515 m **d)**10 s **e)** 15.874 s **f)**20.15 s
2)a) \$0.20 **b)** \$2.20 **c)** \$484 000
3) 5 m
4) 12 cm and 5 cm
5) base 44 cm, height 14 cm
6) 11, 12, 13, 14 or −14, −13, −12, −11
7) width 11 m, length 19 m
8) 16 m